Coursework MPHY0030 2020-2021 Part 2 Report

1. Question 1

The polynomial part is not needed because polynomial part is always used to guarantee the non-singularity of the matrix to get a single solution of the equation. But by using the Gaussian spline, which is positive definite, the non-singularity is already confirmed so there is no need to use a polynomial part.

1. Question 2

Because no polynomial part is needed, the linear algebra for Gaussian spline becomes:

If there is no expected accuracy of the landmarks, then we can set W to an identity matrix, so

is the prediction of target points. To get the solution of , we need to make sure the difference between and is smallest. We would use least square to solve it, that is

To get the minimum, the derivative of should be computed and set the result to 0. So, if we set , then

However, under our circumstances, the problem is that could be a very huge matrix so that the total computation will be very slow. So, we will use the singular value decomposition (SVD) method to solve this least square problem.

For any matrices, such as , it could be decomposed to

in which , , , ,are both singular vectors of , is , is a diagonal matrix, which values are singular values of , so

could be written as , then

The equal sign exists only when , which means

So,

This is exactly the solution of this least square problem.

1. Question 3

There are several linear algebra algorithms to solve this spline fitting problem, such as SVD, QR decomposition, LU decomposition, etc. When using them to solve a least square problem, the best one is SVD because LU always needs the above is inversible, QR is faster than SVD but less stable.

1. Question 4

Control points are points extracted from the query points. They are part of the query points. We cannot choose any points at evaluate stage, because they must correspond to the points at fit stage, which means they should be the same as the source points.

1. Question 5

We do not need the weighting parameter lambda at evaluate stage, because lambda is used to offset the approximated localization errors but there are no localization errors at evaluate stage. The points we use at fit stage are extracted from the initial query points so there are always some errors between the actual points and the points we localized. But at evaluate stage, the points we use are exactly the query points, which do not have localization errors.

1. Question 6

When the data set is very large, it is faster to compute K directly by matrices computing rather than using loop. The following is my vectorization strategy.

First, we suppose the query points’ size is and control points’ size is . The distance will be a matrix. The i row of query points is , the j row of control points is . The distance squared between and is

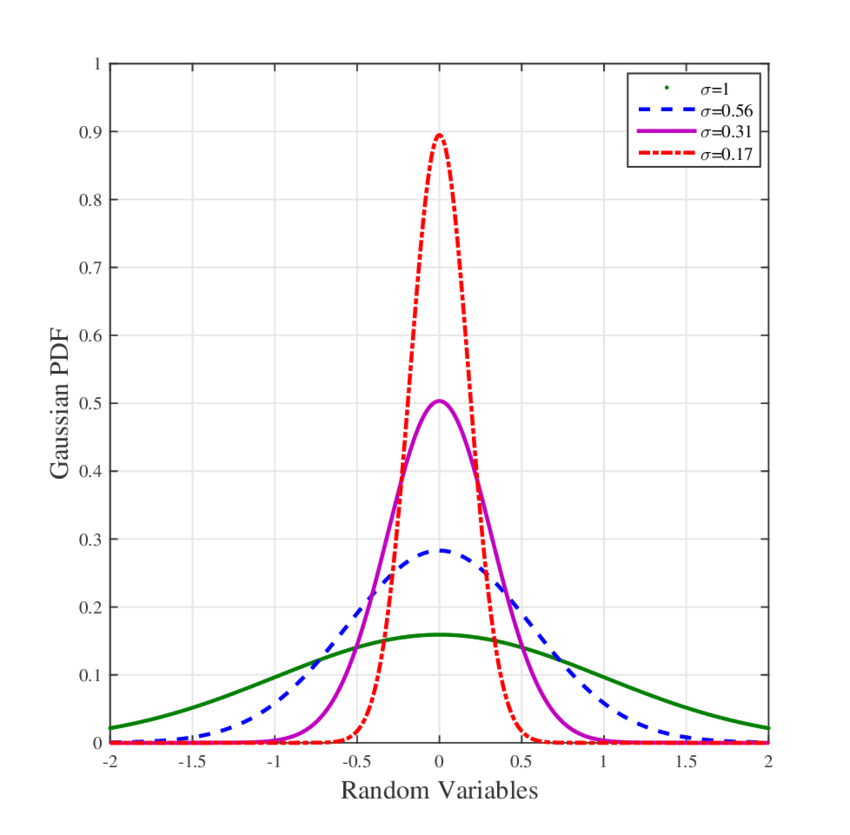
Then it is possible to extend the formula above to the i row of distance matrix. That is

Naturally, we can extend the formula above to the whole distance matrix, which is

Thus, we complete the vectorization method of computing the squared distance matrix.

1. Question 7

The Gaussian kernel parameter sigma indicates a shape parameter that can be used to scale the input of the radial kernel. The Gaussian kernels under different sigma are showed in Figure 1, in which the Gaussian kernels is higher and “thinner” when sigma decreases. Thus, by using different sigma, the same change of r can have different change of R.



1. Question 8

The control points represent a part of the original points’ lattice space so the voxel area of control points is a part of the original medical figure, which is biophysical. Thus, after changing the positions of control points for a little bit, it should still be biophysically reasonable, which means the relative positions of two points should not be reversed. To obtaining this effect, the random distribution should be of Gaussian shape but the maximum of displacement of each point will have a constraint, which should not extend the distance between any two points.

1. Question 9

Because of the biophysically plausible of control points, and the original points are interpolated by the control points, so the interpolated voxel coordinates also represent biophysically plausible deformation.

1. Question 10

We can compute a warped 3D image by the following steps:

1. Get a set of control points by an initialized object from class FreeFormDeformation.
2. Compute the transformed control points by using the random\_transform\_generator.
3. Fit the pre-transformed control points and transformed control points by using the fit function in the class RBFSpline and get spline coefficient , during which the kernel values between two set of control points are computed by kernel\_gaussian.
4. Initialize an object in class Image3D to represent an image which will be warped. And transform the object to lattice structure points, the query points.
5. Evaluate the query points by applying and the kernel values between query points and control points, then get transformed query points.
6. Convert the transformed query points back to an object, the warped 3D image.