Coursework MPHY0030 2020-2021 Part 2 Report

1. Question 1

The polynomial part is not needed because polynomial part is always used to guarantee the non-singularity of the matrix to get a single solution of the equation. But by using the Gaussian spline, which is positive definite, the non-singularity is already confirmed so there is no need to use a polynomial part.

1. Question 2

Because no polynomial part is needed, the linear algebra for Gaussian spline becomes:

If there is no expected accuracy of the landmarks, then we can set W to an identity matrix, so

is the prediction of target points. To get the solution of , we need to make sure the difference between and is smallest. We would use least square to solve it, that is

To get the minimum, the derivative of should be computed and set the result to 0. So, if we set , then

1. Question 3

There are several linear algebra algorithms to solve this spline fitting problem, such as singular value decomposition (SVD), QR decomposition, LU decomposition, etc. When using them to solve a least square problem, the best one is SVD because LU always needs the above is inversible, QR is faster than SVD but less stable.

1. Question 4

Control points are points extracted from the query points. They are part of the query points. We cannot choose any points at evaluate stage, because they must correspond to the points at fit stage, which means the same size as source points and target points.

1. Question 5

We do not need the weighting parameter lambda at evaluate stage, because lambda is used to offset the approximated localization errors but there are no localization errors at evaluate stage. The points we use at fit stage are extracted from the initial query points so there are always some errors between the actual points and the points we localized. But ate evaluate stage, the points we use are exactly the query points, which do not have localization errors.

1. Question 6

When the data set is very large, it is faster to compute K directly by matrices computing rather than using loop. The following is my vectorization strategy.

First, we suppose the query points’ size is and control points’ size is . The distance will be a matrix. The i row of query points is , the j row of control points is . The distance squared between and is

Then it is possible to extend the formula above to the i row of distance matrix. That is

Naturally, we can extend the formula above to the whole distance matrix, which is

Thus, we complete the vectorization method of computing the squared distance matrix.

1. Question 7

The Gaussian kernel parameter sigma indicates a shape parameter that can be used to scale the input of the radial kernel. The Gaussian kernels under different sigma are showed in Figure 1, in which the Gaussian kernels is higher and “thinner” when sigma decreases. Thus, by using different sigma, the same change of r can have different change of R.

